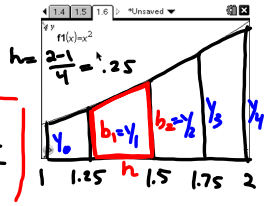
5.5 Trapezoidal Rule

Approximate the area under y=x^2 from 1 to 2 using trapezoids.



area of a trapezoid $= (b_1+b_2)\frac{h}{2}$

$$\text{Trap} = (y_0+y_1)\tfrac{h}{2} + (y_1+y_2)\tfrac{h}{2} + (y_2+y_3)\tfrac{h}{2} + (y_3+y_4)\tfrac{h}{2}$$
$$= \tfrac{h}{2}\left[y_0+y_1+y_1+y_2+y_2+y_3+y_3+y_4\right]$$
$$= \tfrac{h}{2}\left[y_0+2y_1+2y_2+2y_3+y_4\right]$$
$$= \tfrac{.25}{2}\left[1^2+2(1.25)^2+2(1.5)^2+2(1.75)^2+2^2\right]$$
$$= 2.344$$

Nov 13-6:01 PM

Trapezoidal Rule:

general rule: $\frac{h}{2}\left[y_0+2y_1+2y_2+\ldots+2y_{n-1}+y_n\right]$

average of LRAM and RRAM:

$$\text{LRAM } \left[y_0+y_1+y_2+\ldots+y_{n-1}\right]h$$
$$\text{RRAM } + \left[y_1+y_2+\ldots+y_{n-1}+y_n\right]h$$
$$\frac{L+R}{2} = \left[y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right]\frac{h}{2}$$

Nov 13-6:39 PM

Ex 2: An observer measures the outside temperature every hour from noon until midnight, recording the temperatures in the following table:

time	N	1	2	3	4	5	6	7	8	9	10	11	M
temp	63	65	66	68	70	69	68	68	65	64	62	58	55

What was the average temperature for the 12-hour period?

$$T = \left[63+2\cdot65+2\cdot68+\ldots+2\cdot58+55\right] = 782$$

$$\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx \approx \frac{1}{b-a}\,\text{Trap}$$

$\frac{782}{12-0} = 65.1667$

"other" average $= \left[63+65+68+\ldots+58+55\right]\frac{1}{13}$
$= 64.69$

Nov 13-6:42 PM

Error bound for Trapezoidal Rule

$$|E_T| \le \left(\frac{b-a}{12}\right)h^2(\text{max of } f'')$$
$$|E_T| \le \left(\frac{b-a}{12}\right)\left(\frac{b-a}{n}\right)^2 M_{f''}$$

Estimate the error in approximating $\int_1^2 \frac{1}{x}\,dx$ with trap(10)

$$|E_T| \le \left(\frac{2-1}{12}\right)\left(\frac{2-1}{10}\right)^2\cdot 2 = \frac{1}{12}\cdot\frac{1}{100}\cdot 2 = \frac{1}{600}$$

$f = \frac{1}{x}$
$f' = -1/x^2$
$f'' = \frac{2}{x^3}$

max of $\frac{2}{x^3}$ on $[1,2] \stackrel{?}{=} 2$

f'' : $x=1$ $x=2$

Nov 13-6:49 PM